A MAGNETOGASDYNAMIC POWER GENERATION STUDY

by

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INTRODUCTION

During the first quarter, a 48 channel oscillograph system has been ordered and is currently being hooked-up and checked out. This will enable all the data (pressures, temperatures, magnetic field, induced voltages, induced currents, pre-ionization power, etc.) to be recorded quickly and continuously. During this time experiments have been conducted in order to "calibrate" the microwave system with an equilibrium plasma. Typical results are shown in Figures 1 and 2. Calibration is satisfactory.

Also during this period the following calculations were performed to determine:

1. The optimum seed concentration in a non-equilibrium plasma and the effect of a variable seed injection rate.
2. Maximum temperature depression due to seeding, taking into account vaporization, heat of seed gas, ionization, and resonance radiation loss.
3. Electron-beam efficiencies.

The discussions of these calculations follow.

THEORETICAL OPTIMUM SEED CONCENTRATIONS IN SLIGHTLY IONIZED NON-EQUILIBRIUM PLASMAS

The electrical conductivity of a gas can be greatly increased by the addition of a small amount of an easily ionized "seed," and also by the creation of a non-equilibrium condition of the electrons. The creation of such non-equilibrium conditions (where the average electron temperature is higher than the average neutral particle temperature) is being widely studied in near atmospheric pressure plasmas. In the field of "closed cycle MHD power generation," the creation of such a non-equilibrium plasma is essential. Since the electron elastic collision cross-section is usually much greater for the seed (typically an alkali metal) than for the parent gas (typically a noble gas), the increase in electron density can be offset by the increase in electron collision frequency when too much seed is added. Rosa (see Reference 2) has briefly discussed the optimum seed concentration in an equilibrium plasma. Russians Zimin and Popov (see References 6 and 7) have greatly extended the analysis of the optimum composition of a gas mixture, also at equilibrium. Presented below is the derivation of an expression for the seed concentration which gives rise to the maximum electrical conductivity in a slightly ionized non-equilibrium plasma. Although the analysis is strictly valid only when a very small amount of the seed is ionized, the values of optimum seed concentration and corresponding conductivity are in error by at most 5% when even 10% of the seed is ionized.

The various species are assumed to possess Maxwellian velocity distributions. Radiation losses are neglected.

Equations

The total plasma electrical resistivity is taken to be the sum of contributions due to electron-atom collisions and electron-ion collisions (see Reference 1).

$$\eta_p = \eta_a + \eta_i \tag{1}$$

where η_p is the total plasma resistivity

η_a is the electron-atom contribution to the plasma resistivity

η_i is the electron-ion contribution to the plasma resistivity

The resistivity due to electron-atom collisions is given by the hard sphere Maxwellian model (see References 3 and 4).

$$\eta_a = \frac{\left(\frac{8\ m_e k T_e}{\pi}\right)^{\frac{1}{2}} \left(n_s Q_{es} + n_n Q_{en}\right)}{e^2 n_e} \tag{2}$$

where m_e is the electron mass

k is the Boltzman constant

T_e is the average electron temperature

n_s is the seed particle density

Q_{es} is the electron-seed atom elastic collision cross-section

n_n is the parent particle density

Q_{en} is the electron-parent atom elastic collision cross-section

e is the electron charge

For convenience of calculations purposes, Equation (2) becomes:

$$\eta_a = 2.21 \times 10^9 \zeta^{\frac{1}{2}} T_n^{\frac{1}{2}} n_e^{-1} (n_s Q_{es} + n_n Q_{en}), \text{ ohm-cm} \tag{3}$$

where ζ is the temperature ratio T_e/T_n

T_e is the electron temperature, ^oK

T_n is the neutral particle temperature, ^oK

n_e is the electron density, electrons/cm^3

n_s is the seed atom density, atoms/cm^3

n_n is the parent atom density, atoms/cm^3

Q_{es} is the elastic electron collision cross-section for the cross-section for the seed, cm^2

Q_{en} is the elastic electron collision cross-section for the parent gas, cm^2

These calculations are performed with constant elastic electron collision cross-sections for the temperature range 1000-3000^{o}K, thus neglecting any Ramsauer effect.

The resistivity due to electron-ion collisions is given by the modified Lorentz gas expression (see References 4 and 5).

$$\eta_i = 6.62 \times 10^3 \zeta^{-\frac{3}{2}} T_n^{-\frac{3}{2}} \ln \Lambda, \text{ ohm-cm} \tag{4}$$

where Λ is the ratio of the Debye shielding length to the average impact parameter

$$\Lambda = 1.2388 \times 10^4 \zeta^{\frac{3}{2}} T_n^{\frac{3}{2}} n_e^{-\frac{1}{2}} \tag{5}$$

The electron density is given by the Saha equation:

$$\frac{n_e^2}{n_s - n_e} = 2.4146 \times 10^{15} \zeta^{\frac{3}{2}} T_n^{\frac{3}{2}} e^{-\frac{11,606\, E_o}{\zeta T_n}}, \quad \frac{\text{electrons}}{\text{cm}^3} \tag{6}$$

where E_o is the ionization potential of the seed, e. v.

Note that only the alkali metal seed is taken to ionize at temperatures less than $3000^{o}K$, and thus the ratio of the statistical weights is taken to be unity.

The ideal gas law is used to determine the number density of parent gas atoms:

$$n_n = 0.734 \times 10^{22} P/T_n, \ \text{atoms/cm}^3 \tag{7}$$

where P is the pressure; atomspheres.

The mole fraction of seed, X, is defined:

$$n_s = X n_n, \ \text{atoms/cm}^3 \tag{8}$$

For the case of a slightly ionized plasma, $n_e << n_s$, the above equation may be combined to yield an explicit expression of the mole fraction of seed which produces the maximum electrical conductivity in a seeded gas plasma.

Under the conditions $n_e << n_s$, Equations (7) and (8) may be substituted into Equation (6) to yield:

$$n_e \simeq 4.210 \times 10^{18} \zeta^{\frac{3}{4}} T_n^{\frac{1}{4}} X^{\frac{1}{2}} P^{\frac{1}{2}} e^{-\frac{5803\, E_o}{\zeta T_n}}, \quad \frac{\text{electrons}}{\text{cm}^3} \tag{9}$$

Substituting Equations (7), (8) and (9) into Equations (1), (3) and 4 yields:

$$\eta_p = 3.853 \times 10^{12} \zeta^{-\frac{1}{4}} T_n^{-\frac{3}{4}} P^{\frac{1}{2}} e^{\frac{5803 E_o}{\zeta T_n}} X^{-\frac{1}{2}} (XQ_{es} + Q_{en})$$

$$+ 6.62 \times 10^{3} \zeta^{-\frac{3}{2}} T_n^{-\frac{3}{2}} \ln \left[6.037 \times 10^{-5} \zeta^{\frac{9}{8}} T_n^{\frac{11}{8}} P^{-\frac{1}{4}} X^{-\frac{1}{4}} \right.$$

$$\left. e^{\frac{2902 E_o}{T_n}} \right], \text{ ohm-cm} \tag{10}$$

For more convenience Equation (10) is rearranged:

$$\eta_p = \alpha X^{-\frac{1}{2}} (XQ_{es} + Q_{en}) + \beta - \gamma \ln X, \text{ ohm-cm} \tag{11}$$

$$\text{where } \alpha \equiv 3.853 \times 10^{12} \zeta^{-\frac{1}{4}} T_n^{-\frac{3}{4}} P^{\frac{1}{2}} e^{\frac{5803 E_o}{\zeta T_n}} \tag{12}$$

$$\beta \equiv 6.62 \times 10^{3} \zeta^{-\frac{3}{2}} T_n^{-\frac{3}{2}} \ln \left[6.037 \times 10^{-5} \zeta^{\frac{9}{8}} T_n^{\frac{11}{8}} P^{-\frac{1}{4}} \right.$$

$$\left. e^{\frac{2902 E_o}{\zeta T_n}} \right] \tag{13}$$

$$\gamma \equiv 1.655 \times 10^{3} \zeta^{-\frac{3}{2}} T_n^{-\frac{3}{2}} \tag{14}$$

In order to obtain the minimum plasma resistivity with respect to variable seed concentration, Equation (11) is differentiated with respect to X and the resulting expression set equal to zero. Thus,

$$\frac{\partial \eta_p}{\partial X} = \frac{\alpha}{2} Q_{es} X_o^{-\frac{1}{2}} - \frac{\alpha}{2} Q_{en} X_o^{-\frac{3}{2}} - \gamma X_o^{-1} = 0 \qquad (15)$$

where X_o is the optimum mole fraction of seed.

Solving for X_o:

$$X_o = \left[\frac{\varphi + \left(\varphi^2 + \frac{4\, Q_{en}}{Q_{es}}\right)^{\frac{1}{2}}}{2} \right]^2 \qquad (16)$$

$$\text{where } \varphi \equiv \frac{2\gamma}{\alpha\, Q_{es}} = 8.61 \times 10^{-10} \zeta^{-\frac{5}{4}} T_n^{-\frac{3}{4}} P^{-\frac{1}{2}} Q_{es}^{-1} e^{-\frac{5803\, E_o}{\zeta T_n}} \qquad (17)$$

This X_o is the required mole fraction to give the optimum non-equilibrium conductivity. Because of the strong temperature dependence in the exponential term in α, for low T_e the term φ in Equation (16) becomes negligible; in this limit Equation (16) reduces to the well known equilibrium expression for low temperatures (see Reference 2);

$$X_o = Q_{en}/Q_{es} \qquad (18)$$

Equation (15) does indeed represent a minimum value of η_p since

$$\left(\frac{\partial^2 \eta_p}{\partial X^2} \right)_{X_o} > 0.$$

Calculations show the negative root omitted in Equation (16) to be extraneous, see Figure 3.

For convenience of computation, the logarithmic form of Equation (17) is presented:

$$\log_{10}\varphi = 9.0659 - \frac{5}{4}\log_{10}\zeta - \frac{3}{4}\log_{10} T_n - \frac{1}{2}\log_{10} P$$

$$- \log_{10} Q_{es} - \frac{2520.2\, E_o}{\zeta\, T_n} \qquad (19)$$

In order for the analysis to be valid and to thus yield values of σ_p and X_o which are better than 95% accurate, the following criterion should be followed:

$$\frac{n_e}{n_s} < 0.1 \qquad (20)$$

or

$$(\zeta T_n)^{\frac{5}{4}} (\zeta X)^{-\frac{1}{2}} P^{-\frac{1}{2}} e^{-\frac{5803\, E_o}{\zeta T_n}} < 174.3 \qquad (21)$$

A typical criterion graph is shown in Figure 4. Calculations involving the values of ζ, X, T_n, the combinations which lie in the lower left corner of the Criterion Graph, are the most accurate. The analysis becomes less valid the further up and to the right the values lie on the criterion graph; however, it should be noted that the analysis is at least 95% accurate for most practical seeded plasma applications such as in MHD power generation and in high pressure diodes.

Discussion

Various equilibrium gas systems at the optimum seed mole fraction, are compared with pure alkali metal vapors in Figure 5. In general, cesium seeded gas systems can yield much higher electrical conductivities than other seeded gas systems or pure alkali metal vapors at the same pressure and temperature. The physical constants used during the calculations are given in Table I. For non-equilibrium electron temperatures, the conductivities are of course much higher. A typical plot of optimum seed concentrations vs. temperature is shown in Figure 6 for an argon-cesium plasma. The equilibrium limit-curve is in agreement with the results presented in Reference 6. For some cases of practical interest, the optimum mole fraction of seed can be an order of magnitude higher than the low temperature equilibrium value. During an actual experiment, the seeding mole fraction will actually vary (probably about some mean value such as the optimum value); the influence of such variations upon the conductivity can be determined from a plot of σ_p vs X (see Figure 7). For a cesium seeded argon plasma, with $1000^{\circ}K < T_n < 2600^{\circ}K$ and with $1.0 < \zeta < 1.3$, as much as a 50% variation in seed mole fraction about the optimum value will give rise to no more than a 15% reduction in the electrical conductivity. This is shown for one case in Figure 7. For the above mentioned range, in seeded gas non-equilibrium and equilibrium plasmas, the conductivity rises sharply with increasing seed concentration until the optimum point is reached; then the conductivity decreases linearly with increasing seed concentration. For increasing values of $X > X_0$, the linear decrease in σ_p is accompanied by an approximately linearly increase in n_e (see Figure 8). (The decrease in σ_p is due to the overpowering influence of the increasing coulomb cross-section.) It should also be noted from Figure 8 that it is safer to over-seed than to under-seed; that is, a variation of σ_p due fluctuations in X when $X < X_o$ will be much larger than when $X \geq X_o$.

MAXIMUM TEMPERATURE DEPRESSION DUE TO SEED INJECTION

Herein developed is a simple model which allows a quick calculation to the maximum temperature depression possible due to vaporization of cold seed, heating of cool seed gas, partial ionization of the seed, and line radiation lost from the seed resonance levels. Experimental depressions in temperature have been observed as shown in Figure 9.

Temperature Depression Due to Loss of Line Radiation

Let Ψ be the total rate of energy loss, due to line radiation, from an optically thin plasma. Let

$$\Psi = \sum_j \sum_{i \neq j} \varphi_{ij} \tag{22}$$

where $\varphi_{i,j}$ represents the contribution from the i^{th} to the j^{th} level. Only spontaneous (downward) transitions will be considered. Thus,

$$\Psi = \sum_j \sum_{i > j} \varphi_{ij} \tag{23}$$

Since the plasma is near thermal equilibrium and at temperatures less than 2000^oK, only the first two levels (resonance at 8521 Å and 8943 Å) will be considered. Population of the upper states is negligible as shown in Figure 10. The population density ratio of the first two levels as a function of temperature, is shown in Figure 11. The rate of energy emitted from a unit volume of plasma which is optically thin, in which transitions take place from the excited states 1 and 2 to the ground state 0, is given by the following expression:

$$\Psi = n_1 A_{10} E_{10} + n_2 A_{20} E_{20} \tag{24}$$

where n_1 is the population density of the 1st excited level

A_{10} is the reciprocal mean life of the 1st excited level

E_{10} is the energy of the 1st excited level with respect to the ground level

n_2 is the population density of the 2nd excited level

A_{20} is the reciprocal mean life of the 2nd excited level

E_{20} is the energy of the 2nd excited level with respect to the ground level

Unit check: $\left(\dfrac{\text{number of atoms in level}}{\text{cm}^3}\right)\left(\dfrac{\text{transitions}}{\text{atom in level-sec.}}\right)\left(\dfrac{\text{energy}}{\text{transition}}\right) = \dfrac{\text{energy}}{\text{cm}^3\text{- sec}}$

Note that induced transitions have been neglected (see Reference 8). This is indicated on the energy diagram shown in Figure 12. The terms A_{10} and A_{20}, sometimes called Einstein coefficients, are related to the oscillator strength by the following expression (see Reference 8):

$$A_{10} = \frac{.6669 \times 10^8}{\lambda_1{}^2} \left(\frac{g_0}{g_1}\right) f_{01}, \frac{\text{transitions}}{\text{atoms in 1st level-sec}} \tag{25}$$

$$A_{20} = \frac{.6669 \times 10^8}{\lambda_2{}^2} \left(\frac{g_0}{g_2}\right) f_{02}, \frac{\text{transitions}}{\text{atom in 2nd level-sec}} \tag{26}$$

where λ_1 is the wave length associated with the first

λ_2 is the wave length associated with the second excited level, in microns

g_0 is the statistical weight of the ground level (2J +1)

g_1 is the statistical weight of the first level (2J +1)

g_2 is the statistical weight of the second level (2J +1)

J is the quantum number representing the total angular momentum of the electrons (see Reference 9)

f_{01} is the oscillator strength associated with the first level

f_{02} is the oscillator strength associated with the second level

The oscillator strengths associated with the energy levels of the cesium atom have been recently reported in Reference 10. The terms n_1 and n_2 in Equation (24) are determined from the Boltzmann expression and the ideal gas law:

$$n_1 = 0.734 \times 10^{22}\, PT^{-1} X_s \left(\frac{g_1}{g_0}\right) e^{-11,606\, E_{10}/T}, \quad \frac{\text{atoms in 1st level}}{\text{cm}^3} \tag{27}$$

$$n_2 = 0.734 \times 10^{22}\, PT^{-1} X_s \left(\frac{g_2}{g_0}\right) e^{-11,606\, E_{20}/T}, \quad \frac{\text{atoms in 2nd level}}{\text{cm}^3} \tag{28}$$

where P is the total pressure, in atmospheres

T is the static temperature, in $^{\circ}$K

X_s is the mole fraction of seed

E_{10} is the energy associated with the 1st level, in e. v.

E_{20} is the energy associated with the 2nd level, in e. v.

g refers to the statistical weight

Thus Equation (24) becomes:

$$\Psi = \left[0.734 \times 10^{22} P T^{-1} X_s \left(\frac{g_1}{g_0} \right) e^{-11,606\, E_{10}/T} \right]$$

$$\left[\frac{.6669 \times 10^8}{\lambda_1{}^2} \left(\frac{g_0}{g_1} \right) f_{01} \right] E_{10} + \left[.0734 \times 10^{22} P T^{-1} X_s \right.$$

$$\left. \left(\frac{g_2}{g_0} \right) e^{-11,606\, E_{20}/T} \right] \left[\frac{.6669 \times 10^8}{\lambda_1{}^2} \left(\frac{g_0}{g_2} \right) f_{02} \right] E_{20} \tag{29}$$

or

$$\Psi = 4.895 \times 10^{29} P T^{-1} X_s \left[\frac{f_{01} E_{10} e^{-11,606\, E_{10}/T}}{\lambda_1{}^2} \right.$$

$$\left. + \frac{f_{02} E_{20}}{\lambda_2{}^2} e^{-11,606\, E_{20}/T} \right] \tag{30}$$

In the above equation Ψ has the units of e.v./cm^3-sec. Since there are 1.602×10^{-19} joules/e.v. and since there are 4.186 joules/cal., Equation (30) becomes

$$\Psi = 1.873 \times 10^{10} P T^{-1} X_s \left[\frac{f_{01} E_{10} e^{-11,606\, E_{10}/T}}{\lambda_1{}^2} \right.$$

$$\left. + \frac{f_{02} E_{20} e^{-11,606\, E_{20}/T}}{\lambda_2{}^2} \right], \quad \frac{\text{cal.}}{\text{cm}^3\text{-sec}} \tag{31}$$

In order to give an upper limit on the temperature depression observed in a volume of plasma with velocity u, and heat capacity c_p, at a distance l downstream of a reference point, the calculation was made with the population density of the initial temperature. Thus,

$$(T_1 - T_3) = \left(\frac{l}{umc_p}\right) \Psi \quad , {}^oK \tag{32}$$

where l is length in cm

m is the total mole density, moles/cm^3

u is velocity in cm/sec

c_p is the heat capacity $\sim$ 5 cal./mole - ^oK for a perfect gas

T_1 is the initial temperature, ^oK

T_3 is the final temperature at point l downstream of reference, ^oK

Since $m = 1.219 \times 10^{-2} PT^{-1}$ moles/cm^3 from the ideal gas law, Equation (32) becomes:

$$T_1 - T_3 = 1.54 \times 10^{12} \left(\frac{l\,X_s}{uc_p}\right) \left[\frac{f_{01}E_{10}e^{-11,606\,E_{10}/T}}{\lambda_1{}^2} + \frac{f_{02}E_{20}e^{-11,606\,E_{20}/T}}{\lambda_2{}^2}\right], {}^oK \tag{33}$$

where l is distance from reference point, in cm

X_s is the mole fraction of seed

u is the plasma velocity

c_p is the heat capacity

f_{01} is the oscillator strength of 1st level

f_{02} is the oscillator strength of 2nd level

E_{10} is the energy of 1st level, e. v.

E_{20} is the energy of 2nd level, e. v.

T is the static temperature, ^{o}K

λ_1 is the wave length associated with the 1st level, in microns

λ_2 is the wave length associated with the 2nd level, in microns

Temperature Depression Due to Vaporization

The temperature depression due to vaporization of the liquid seed is obtained from the following expression:

$$m_1 c_p (T_1 - T_3) = m_2 \Delta H_2 \tag{34}$$

Thus,

$$(T_1 - T_3) = \frac{m_2 \Delta H_2}{m_1 c_p} \simeq X_s \frac{\Delta H_2}{c_p}, \; ^{o}K \tag{35}$$

where m_1 is the mole density of Argon

c_p is the heat capacity (taken as ideal gas)

m_2 is the mole density of cesium

ΔH_2 is the heat of vaporization per mole of seed

T_1 is the initial temperature, ^{o}K

T_3 is the final temperature, ^{o}K

Since $m_2 << m_1$, $m_2/m_1 + m_2 \approx m_2/m_1 \approx X_s$, the mole fraction of seed. In order to obtain an upper limit, the heat of vaporization is assumed constant at the lowest temperature encountered, see Figure 13.

Temperature Depression Due to Heating of Seed

The temperature depression associated with heating the seed is obtained from the following expression:

$$m_1(T_1 - T_3) = m_2(T_3 - T_2) \tag{36}$$

where m_1 is the mole density of argon

m_2 is the mole density of seed

T_1 is the initial argon temperature

T_3 is the final mixture temperature

T_2 is the initial seed temperature

Note that in Equation (36), the heat capacity of argon is assumed to be equal to that of the seed. Since $m_2 \ll m_1$, Equation (36) becomes:

$$(T_1 - T_3) = X_s (T_3 - T_2) \tag{37}$$

<u>Temperature Depression Due to Partial Ionization of Seed</u>

If ϵ represents the mole fraction of seed ionized, then since $\epsilon \ll 1$ for atmospheric pressure plasmas near 1500°K, ϵ can be approximated from the Saha equation and the ideal gas law as follows:

$$\epsilon = 5.74 \times 10^{-4} T_3^{\frac{5}{4}} X_s^{-\frac{1}{2}} P^{-\frac{1}{2}} e^{-5803\, E_i/T_3} \tag{38}$$

where T_3 is the plasma temperature, °K

X_s is the seed mole fraction

P is the static pressure, in atmospheres

E_i is the ionization potential of the seed

Thus

$$m_1 c_p (T_1 - T_3) = 2.31 \times 10^4 \, \epsilon m_2 E_i \tag{39}$$

where m_1 is the mole density of argon, moles/cm^3

c_p is the heat capacity, cal. /mole $^\circ$K

T_1 is the initial temperature, $^\circ$K

T_3 is the final plasma temperature, $^\circ$K

m_2 is the mole density of seed, moles/cm^3

E_i is the ionization potential of the seed, e. v.

Note:

$$(\text{Moles})\left(\frac{1\ \text{e.v.}}{\text{electron}}\right)\left[\frac{6.02\ \times 10^{23}\ \text{electrons}}{\text{mole}}\right]\left[\frac{1.602\ \times 10^{-14}\ \text{joules}}{\text{e.v.}}\right]\left[\frac{\text{cal.}}{4.186\ \text{joules}}\right]$$

$= 2.31\ \times 10^4$ cal.

Thus, the temperature depression is:

$$(T_1 - T_3) = 1.33\ \times 10^1\ \frac{T_3^{\frac{5}{4}} X_s^{\frac{1}{2}} P^{-\frac{1}{2}}}{c_p}\ E_i e^{-\frac{5803\ E_i}{T_3}} \qquad (40)$$

where T_1 is initial temperature, $^\circ$K

T_3 is final temperature, $^\circ$K

X_s is mole fraction of seed

P is the static pressure, atmospheres

E_i is the ionization potential, e. v.

c_p is the heat capacity, cal. /mole - $^\circ$K

Calculations are performed for the experimental case

where $T_1 = 1500^\circ$K (initial temperature)

$l = 5$ cm

$u = 10^4$ cm/sec

$X_s = 5 \times 10^{-3}$ (Cesium seed in argon)

$T_1 = 1500^\circ K$

$E_{10} = 1.39$ e.v. (First excited level)

$E_{20} = 1.46$ e.v. (Second excited level

$E_i = 3.87$ e.v. (Ionization potential)

$T_2 = 300^\circ K$ (Minimum possible)

$c_p = 5$ cal/mole - $^\circ K$ (Ideal Gas)

$\Delta H_2 = 20,000$ cal./mole (Maximum possible)

$f_{01} = .394$ (See Reference 10)

$f_{02} = .814$ (See Reference 10)

$\lambda_1 = .8521\ \mu$ (See Reference 10)

$\lambda_2 = .8943\ \mu$ (See Reference 10)

The maximum temperature depressions due to the above mentioned mechanisms are listed in Table II. Since this maximum temperature depression model does not account for the observed temperature drops, it is postulated that the seed is entering as a partial liquid spray which impinges upon the thermocouple. At higher temperatures (see Figure 15) more of the spray is vaporized within the injection tube before it enters the duct. Downstream of the point, where the temperature depression is observed, are molybdenum screens through which the flow is forced to pass. Beyond these screens no temperature depression has ever been observed thus indicating that if a depression does exist, it is less than $5K^\circ$.

THEORETICAL ELECTRON BEAM EFFICIENCY

Presented below are preliminary calculations which indicate the maximum possible power increase as a function of energy imparted to the working fluid from the electron beam. Recombination from the point of injection to the magnetic field region and radiation effects are neglected.

For the case where radiative recombination is the primary mechanism for de-ionization, the power input per unit volume, P_i, from the electron beam is:

$$P_i = \alpha n_e^{2} I \tag{41}$$

where α is the recombination coefficient

n_e is the electron density

I is the ion-pair production energy.

The power generated per unit volume in a segmented electrode generator is

$$P_o = \frac{K(1-K)\sigma_p u^2 B^2}{1 + \beta_e \beta_i} \tag{42}$$

where K is the loading factor which is the ratio of the load voltage to the open circuit voltage

σ_p is the plasma electrical conductivity

u is the average plasma velocity

B is the magnetic field strength

$\beta_e \equiv \omega_e \tau_e$

ω_e is the electron cyclotron frequency

τ_e is the average time between electron and non-electron collisions

$\beta_i \equiv \omega_i \tau_i$

ω_i is the ion cyclotron frequency

τ_i is the average time between ion and non-ion collisions.

The electron beam efficiency, η, is defined as follows:

$$\eta = 1 - \frac{P_i}{P_o} \tag{43}$$

For the case K = 1/2 and $\beta_e \beta_i << 1$, the above expression becomes

$$\eta = 1 - \frac{4\,\alpha n_e^2 I}{\sigma_p u^2 B^2} \tag{44}$$

in order for the electron beam to be a practical pre-ionization technique, η 1. For convenience of computation, the following expression is presented:

$$\eta = 1 - \frac{6.41 \times 10^{-3}\,\alpha n_e^2 I}{\sigma_p u^2 B^2} \tag{45}$$

where α is in cm^3/sec

n_e is in $electrons/cm^3$

I is in e.v.

σ_p is mhos/cm

u is in cm/sec

B is in gauss

In order for there to be a significant fraction of the plasma enthalpy to be converted into electrical energy, the magnetic interaction parameter, Q, will be close to unity. For convenience, the following form of the magnetic interaction parameter is presented:

$$Q = 10^{-9} \frac{\sigma_p B^2 L}{\rho u} \tag{46}$$

where σ_p is in mhos/cm

B is in gauss

L is in cm

ρ is in gr/cm^3

u is in cm/sec

Substitution of Equation (46) (for a magnetic interaction parameter of unity) into Equation (45) yields:

$$\eta = 1 - 6.41 \times 10^{-12} \frac{\alpha n_e^2 I L}{\rho u^3} \tag{47}$$

For a typical case of I - 40 e.v., $L = 10^2$ cm, $\rho = 10^{-4}$ gr/cm^3 and $u = 10^5$ cm/sec, Equation (48) becomes:

$$\eta = 1 - 2.56 \times 10^{-19} \alpha n_e^2 \tag{48}$$

The efficiency vs. number density of electrons is presented in Figure 14 for various radiative recombination rates.

TABLE I

Element	Q_e, cm^2	E_o, e.v.
Lithium	2.0×10^{-14}	5.363
Sodium	3.0×10^{-14}	5.12
Potassium	4.0×10^{-14}	4.318
Rubidium	4.7×10^{-14}	4.159
Cesium	5.3×10^{-14}	3.87
Argon	2×10^{-17}	----
Helium	5×10^{-16}	----

TABLE II

Mechanism	ΔT	Equation Used
Resonance Radiation	< 42 K°	# 33
Vaporization	< 20 K°	# 34
Heating of Seed	< 9 K°	# 36
Partial Ionization of Seed	< 1 K°	# 39
Total ΔT	< 72 K°	

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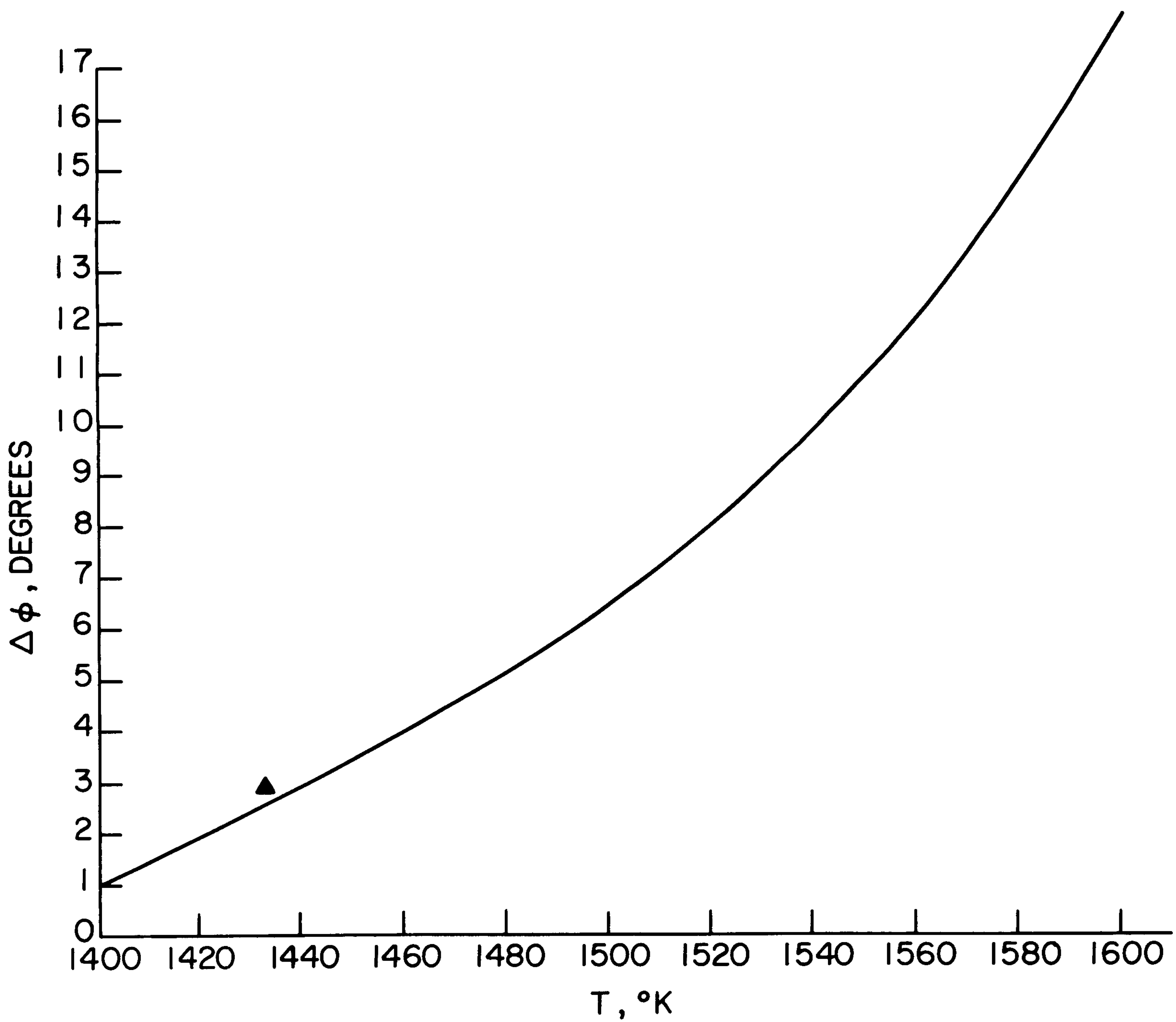


Figure 1. Phase Shift vs Temperature for Equilbrium Plasma

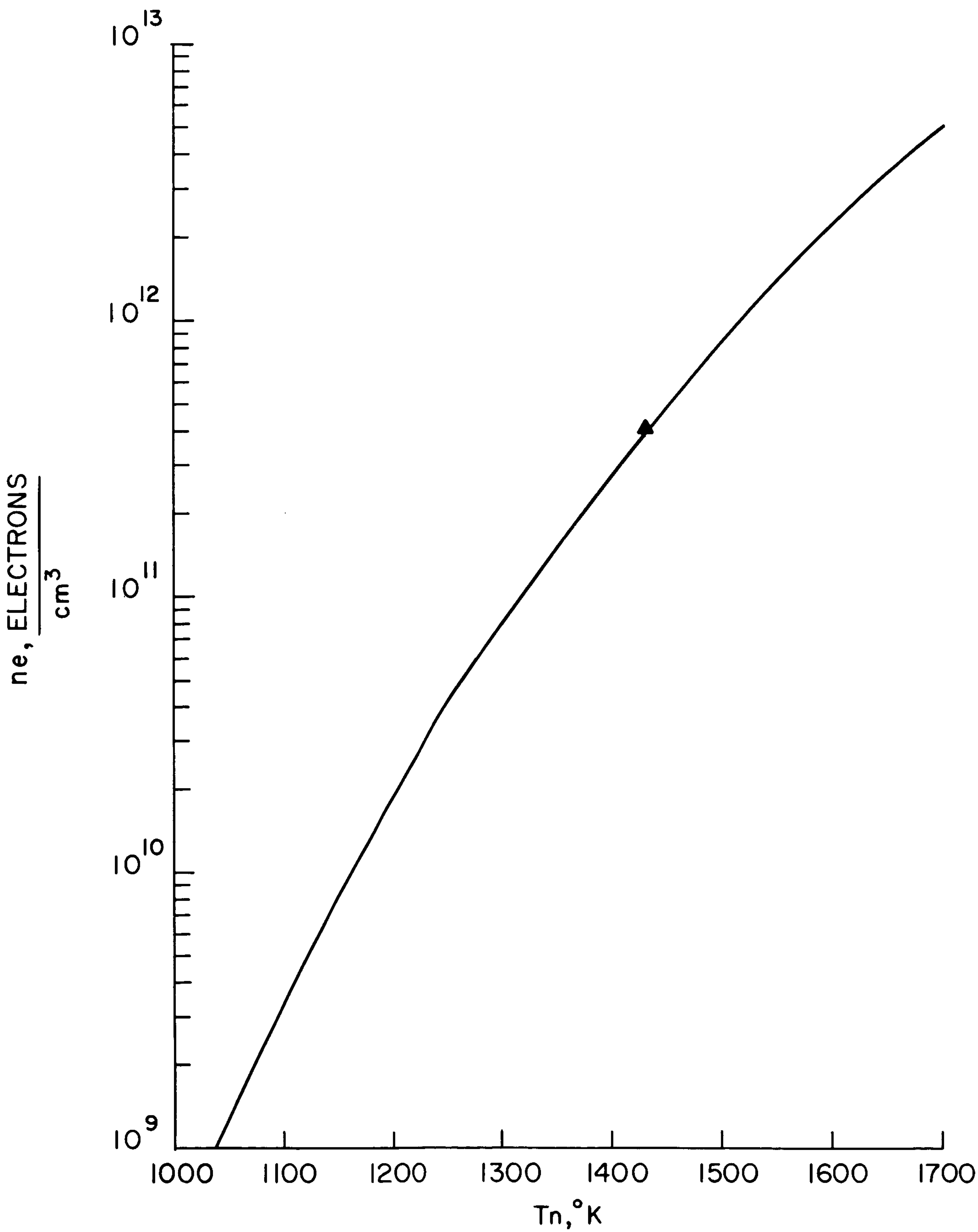


Figure 2. Equilibrium Electron Density vs Temperature for $X=5.2x10^{-3}$, P=2 atm. Cesium Seeded Argon

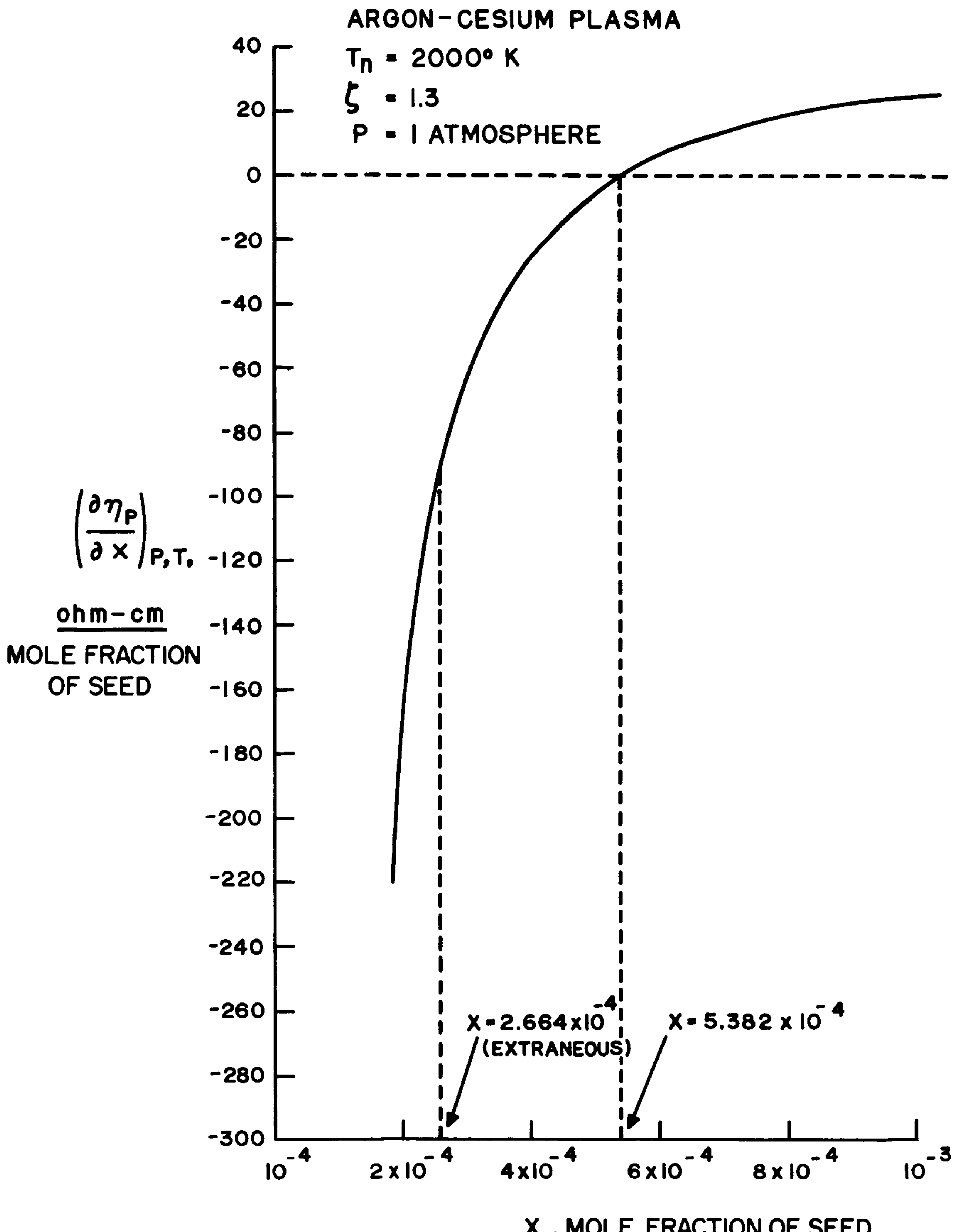

ARGON-CESIUM PLASMA
T_n = 2000° K
ζ = 1.3
P = 1 ATMOSPHERE
$\left(\frac{\partial \eta_P}{\partial X}\right)_{P,T,}$
ohm-cm
MOLE FRACTION OF SEED
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
-200
-220
-240
-260
-280
-300
$X = 2.664 \times 10^{-4}$
(EXTRANEOUS)
$X = 5.382 \times 10^{-4}$
10^{-4}
2×10^{-4}
4×10^{-4}
6×10^{-4}
8×10^{-4}
10^{-3}
X, MOLE FRACTION OF SEED

Figure 3.

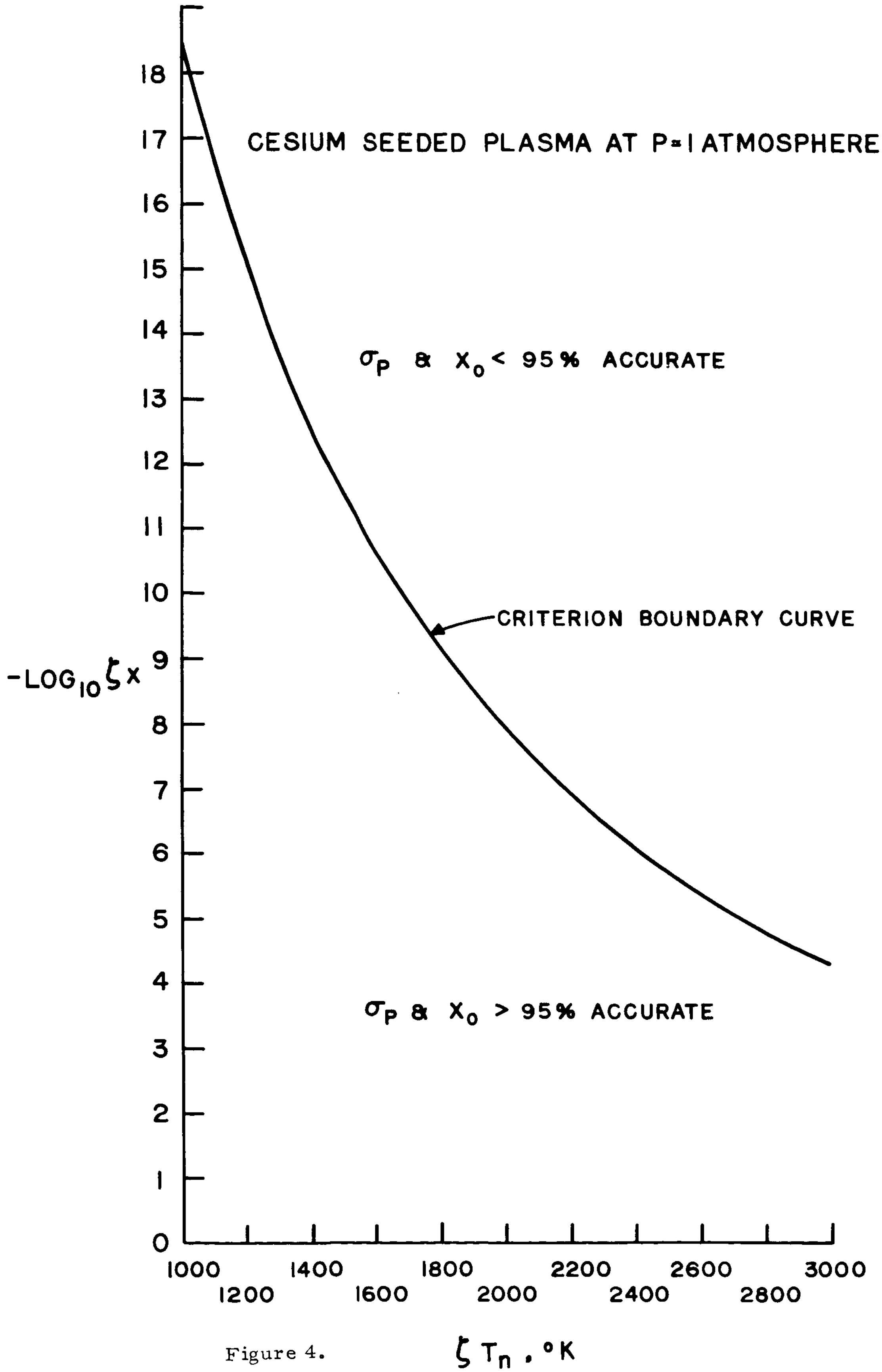
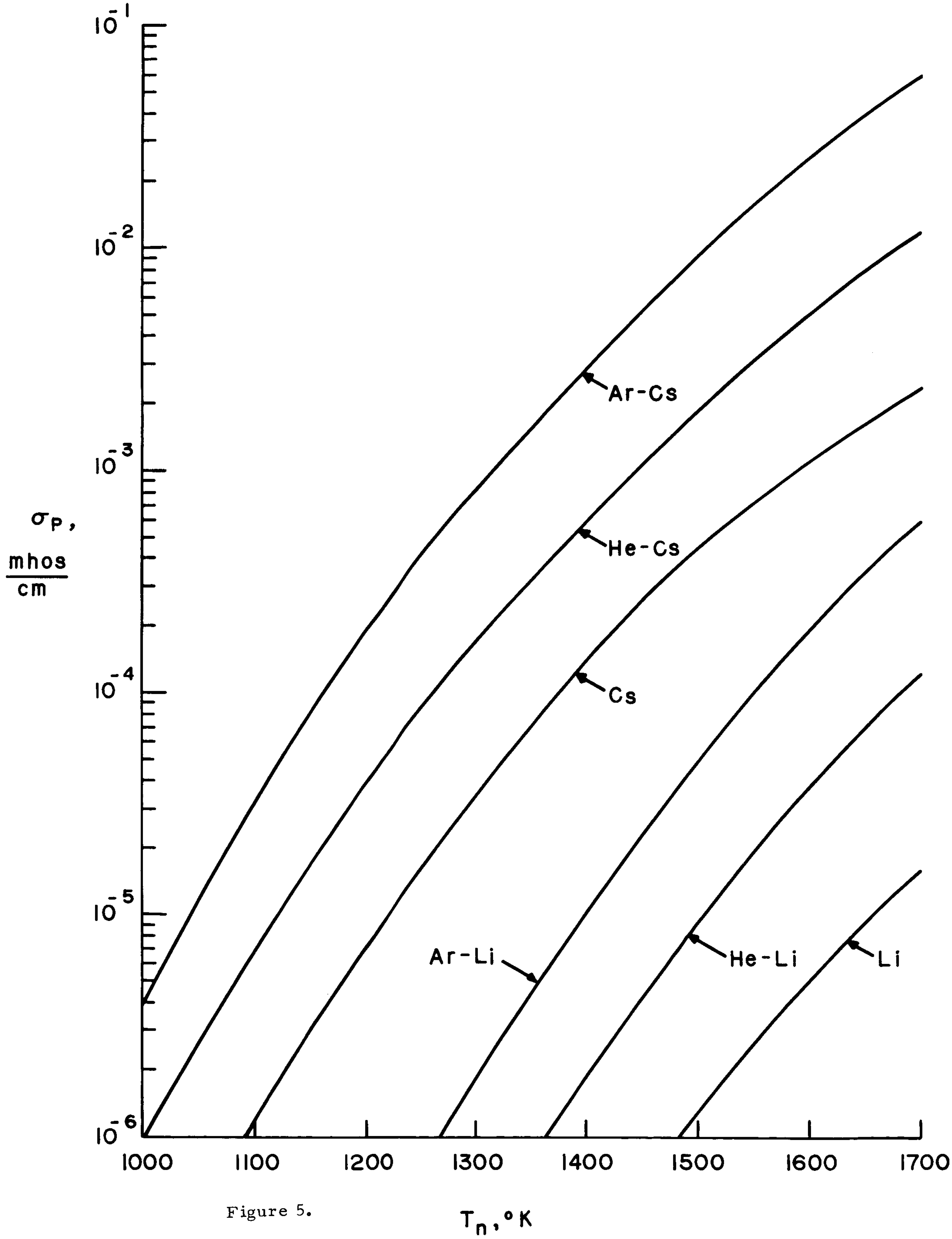
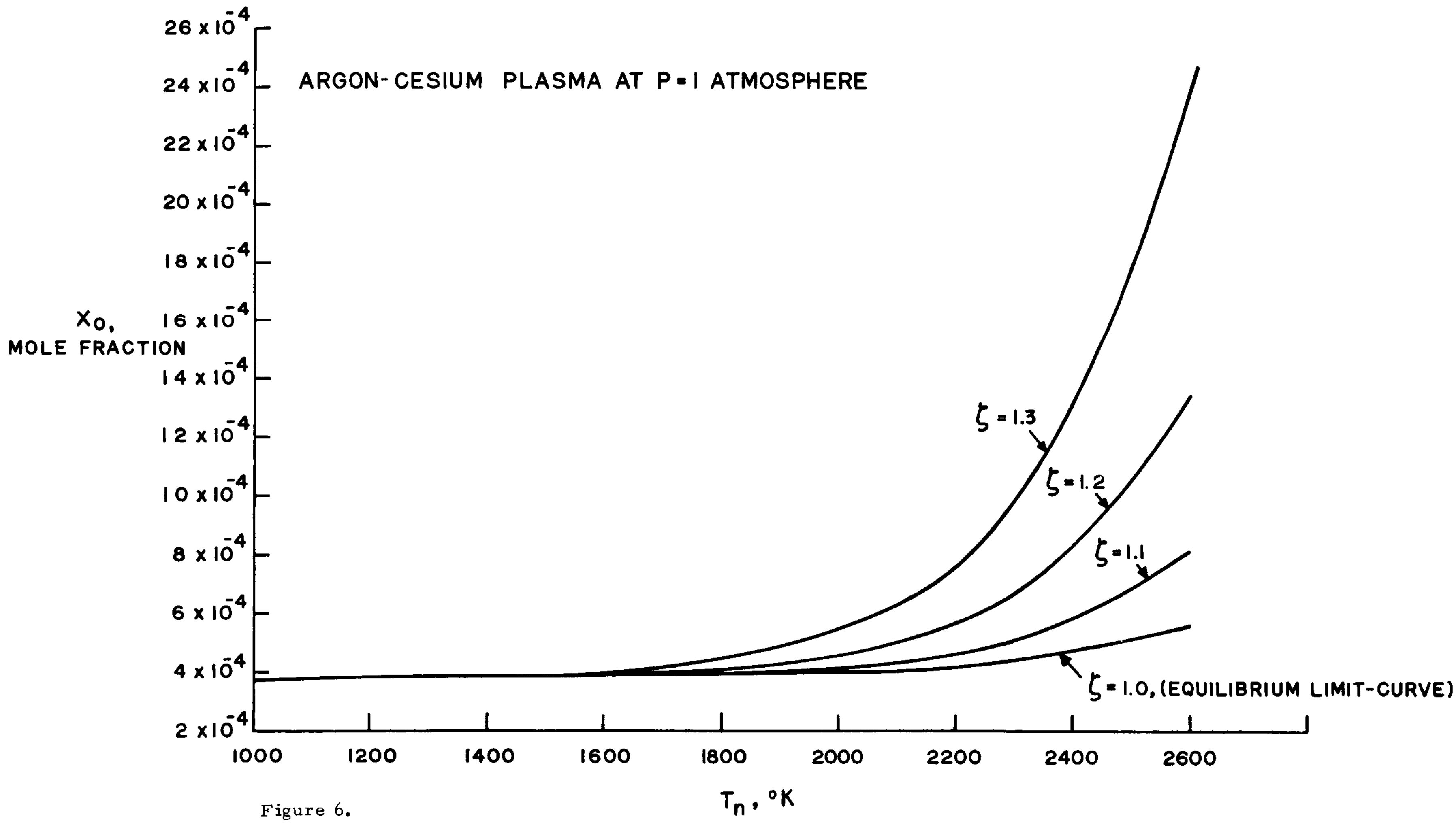

CESIUM SEEDED PLASMA AT P=1 ATMOSPHERE
σ_P & X_0 < 95% ACCURATE
CRITERION BOUNDARY CURVE
σ_P & X_0 > 95% ACCURATE
$-LOG_{10} \zeta x$
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
3000
ζT_n, °K

Figure 4.



10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
σ_P,
$\frac{\text{mhos}}{\text{cm}}$
Ar-Cs
He-Cs
Cs
Ar-Li
He-Li
Li
1000
1100
1200
1300
1400
1500
1600
1700
T_n, °K

Figure 5.



ARGON-CESIUM PLASMA AT P = 1 ATMOSPHERE
26 x 10^-4
24 x 10^-4
22 x 10^-4
20 x 10^-4
18 x 10^-4
16 x 10^-4
14 x 10^-4
12 x 10^-4
10 x 10^-4
8 x 10^-4
6 x 10^-4
4 x 10^-4
2 x 10^-4
X0, MOLE FRACTION
ζ = 1.3
ζ = 1.2
ζ = 1.1
ζ = 1.0, (EQUILIBRIUM LIMIT-CURVE)
1000
1200
1400
1600
1800
2000
2200
2400
2600
Tn, °K

Figure 6.

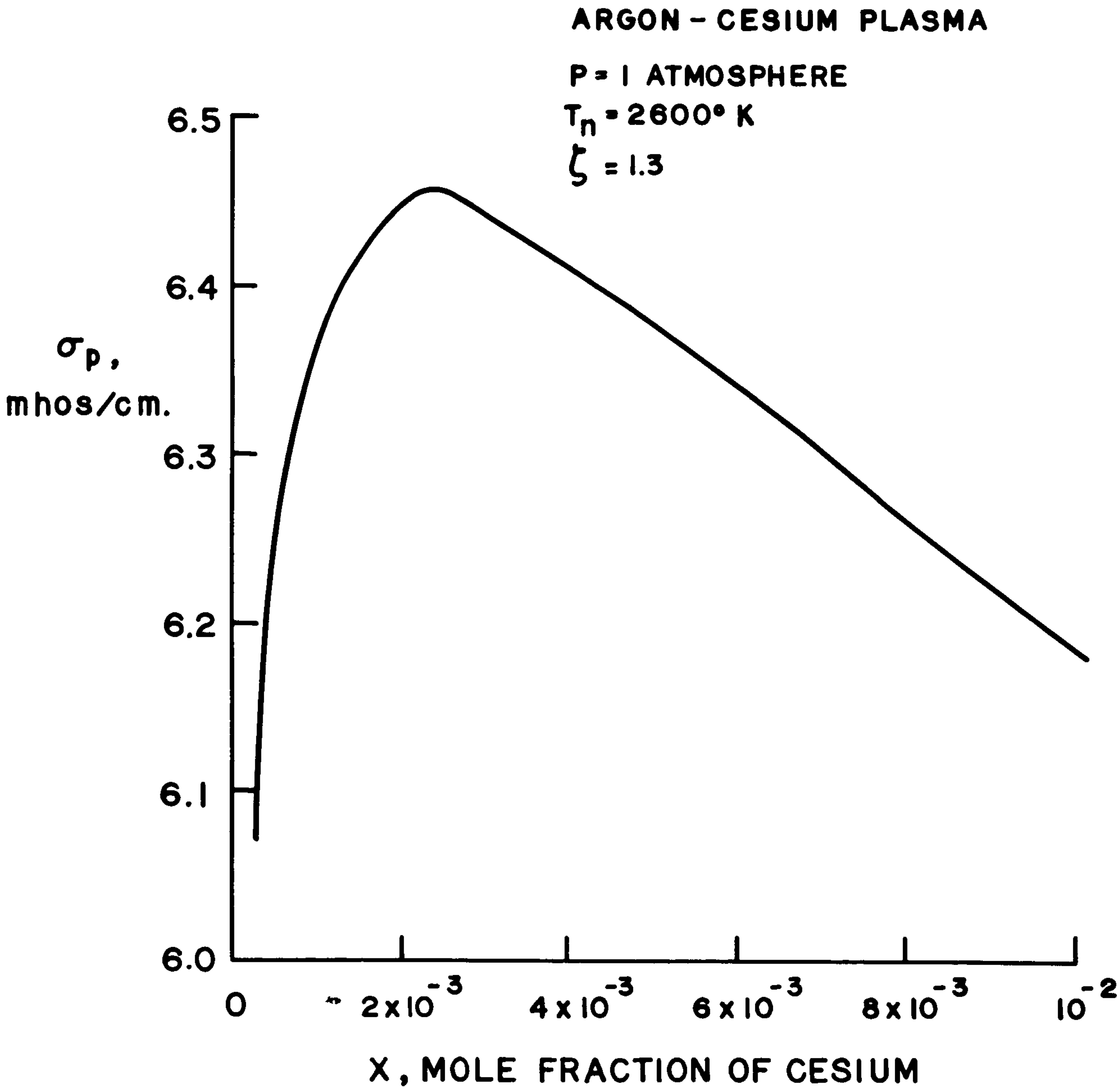

ARGON - CESIUM PLASMA
P = 1 ATMOSPHERE
T_n = 2600° K
ζ = 1.3
6.5
6.4
6.3
6.2
6.1
6.0
σ_p,
mhos/cm.
0
2×10^{-3}
4×10^{-3}
6×10^{-3}
8×10^{-3}
10^{-2}
X, MOLE FRACTION OF CESIUM

Figure 7.

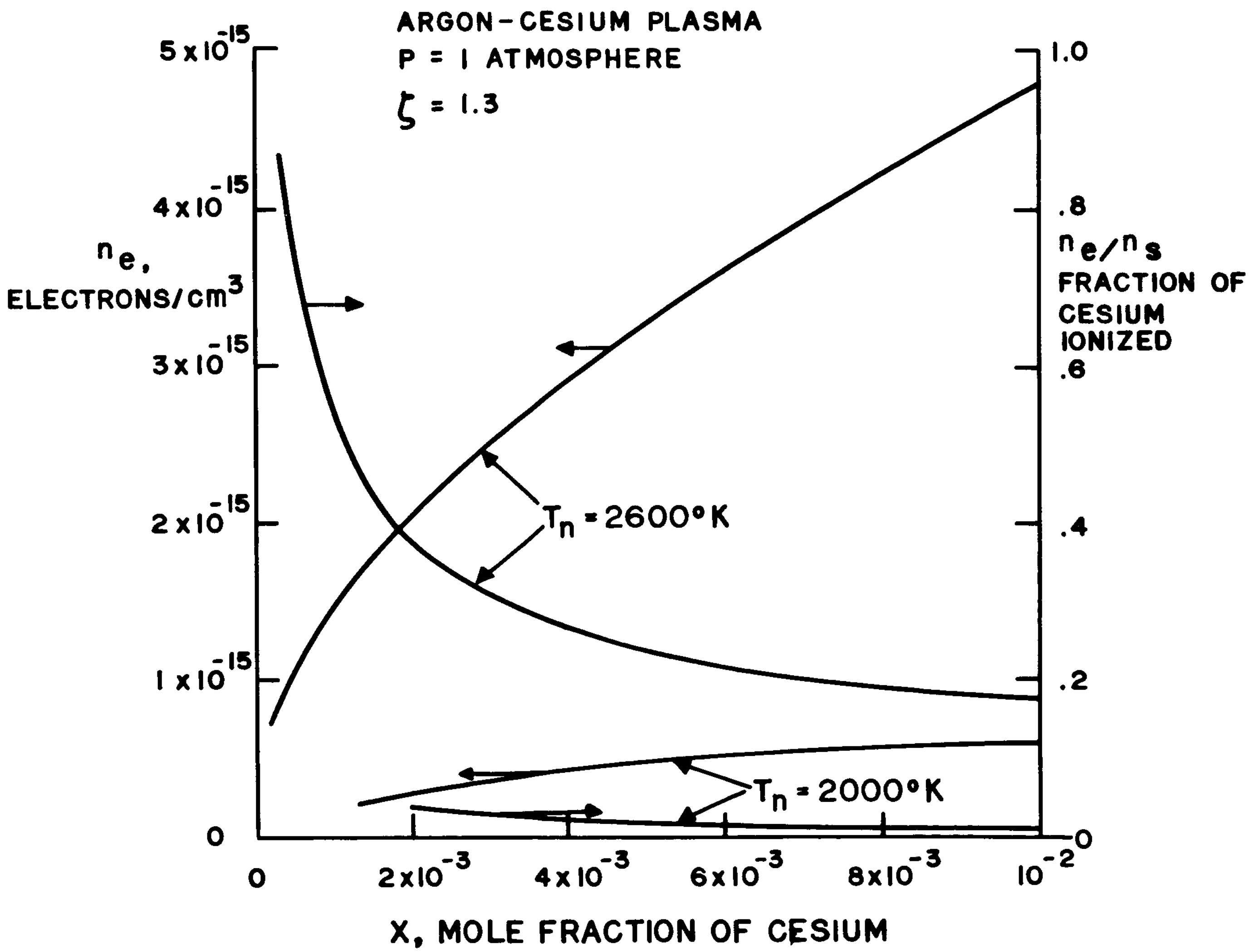

ARGON-CESIUM PLASMA
P = 1 ATMOSPHERE
ζ = 1.3
5 x 10^-15
4 x 10^-15
3 x 10^-15
2 x 10^-15
1 x 10^-15
0
n_e, ELECTRONS/cm^3
1.0
.8
.6
.4
.2
0
n_e/n_s FRACTION OF CESIUM IONIZED
T_n = 2600°K
T_n = 2000°K
0
2x10^-3
4x10^-3
6x10^-3
8x10^-3
10^-2
X, MOLE FRACTION OF CESIUM

Figure 8.

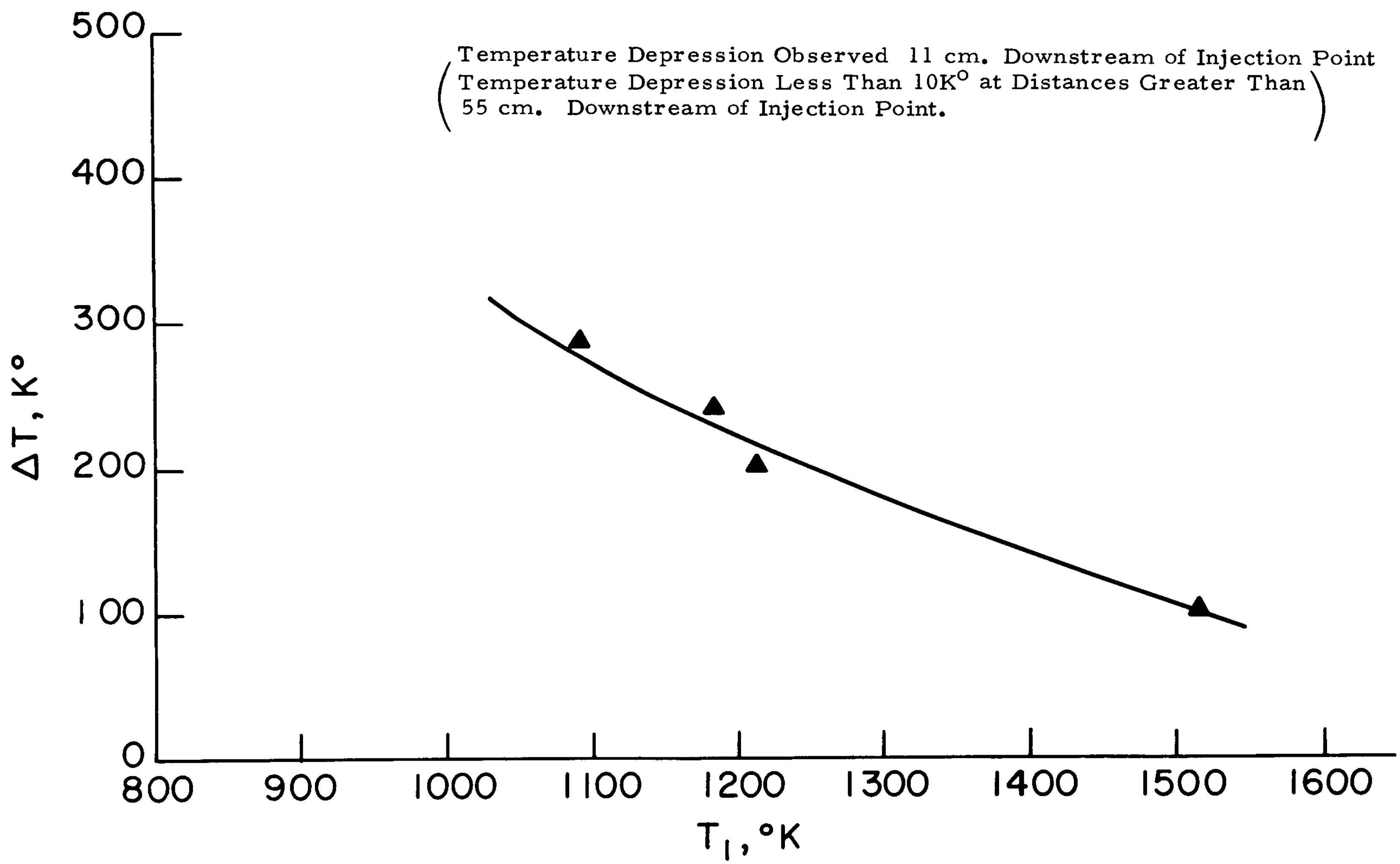


Figure 9. Temperature Depression Due to Seeding vs Initial Gas Temperature

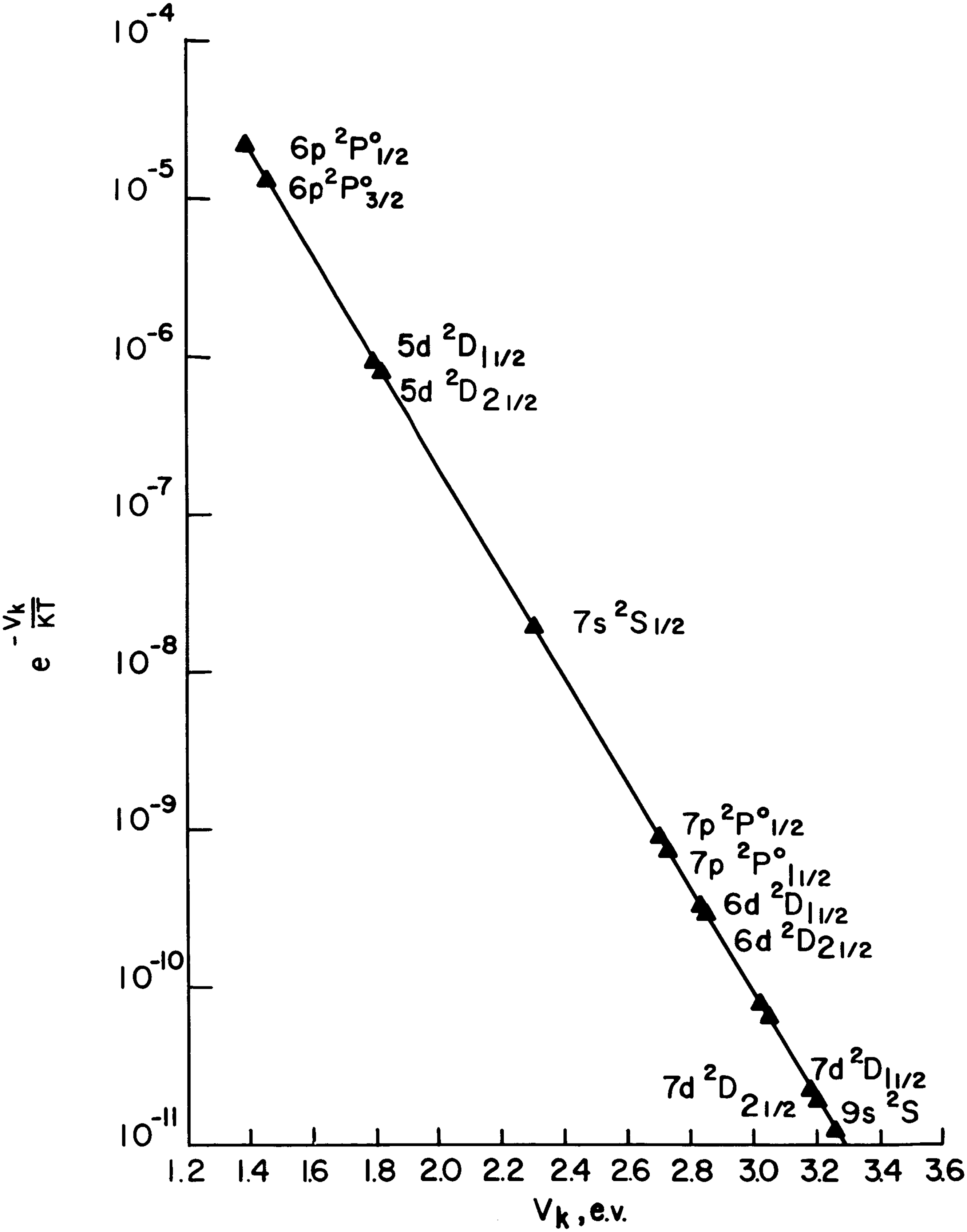


Figure 10. Boltzmann's Factor as a Function of Energy Level for Cesium at $T=1500^{\circ}K$

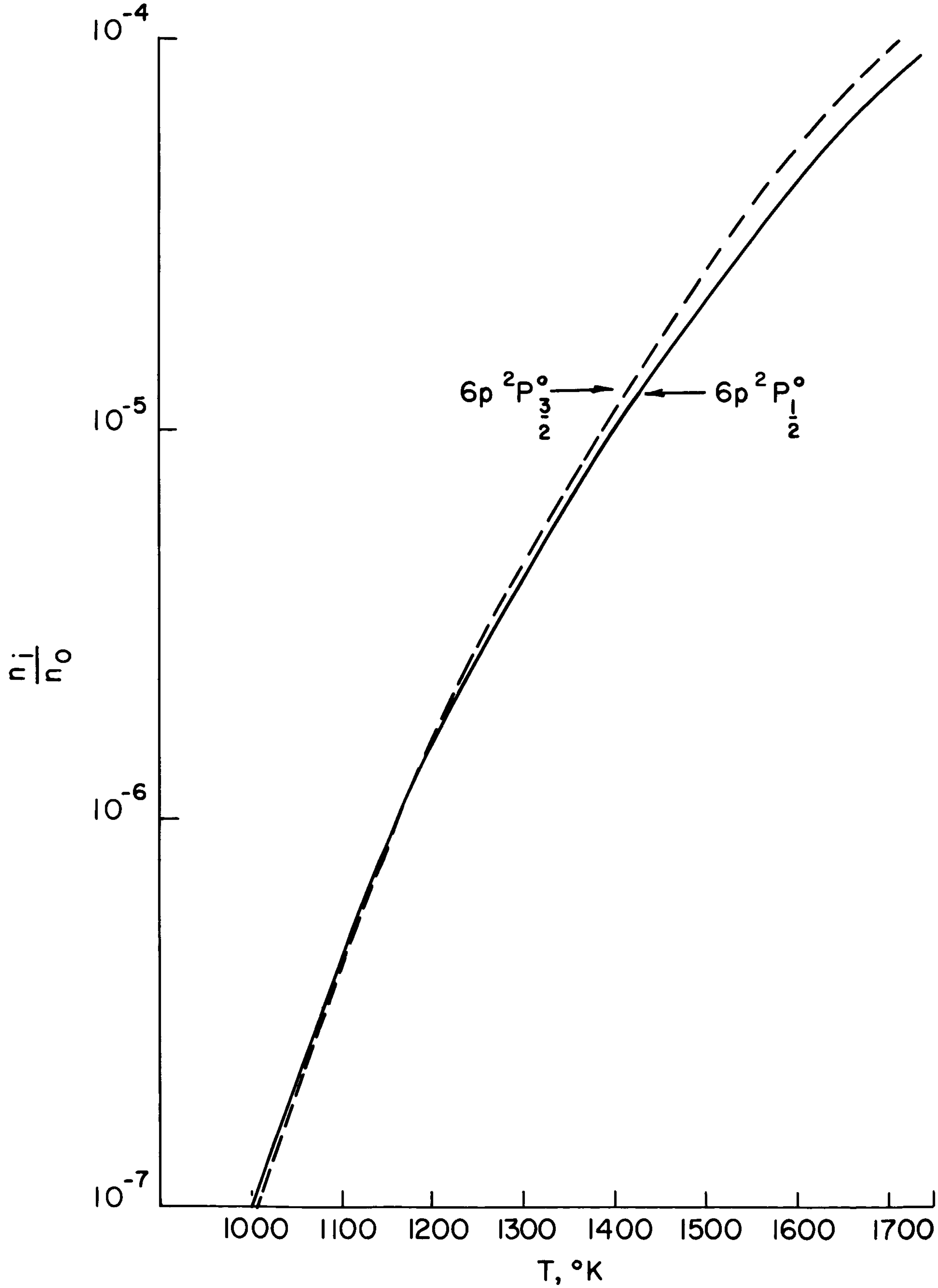


Figure 11. Population vs Temperature For Resonance Levels of Cesium

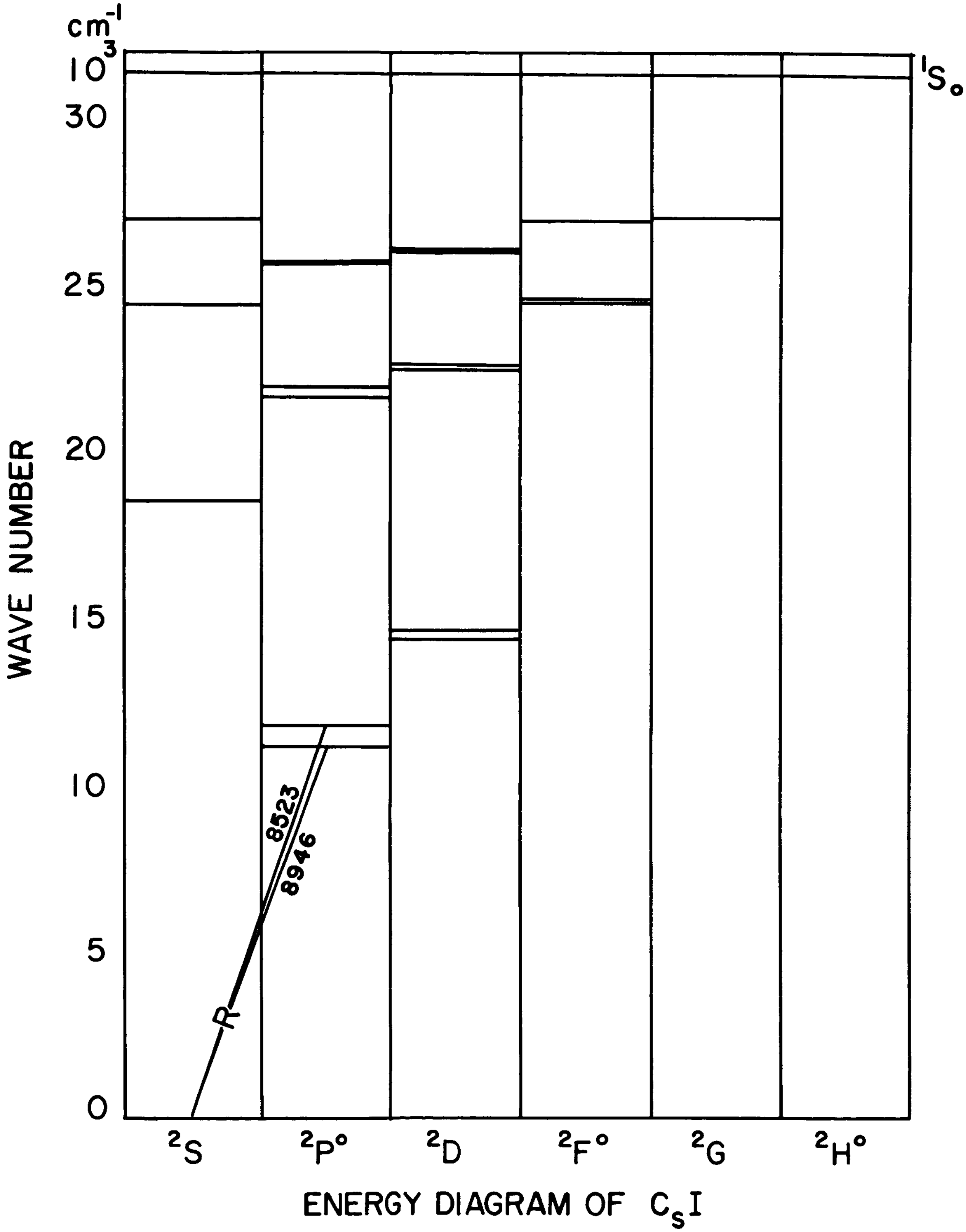

cm-1
10^3
30
25
20
15
10
5
0
WAVE NUMBER
1S0
8523
8946
R
2S
2P°
2D
2F°
2G
2H°
ENERGY DIAGRAM OF CsI

Figure 12.

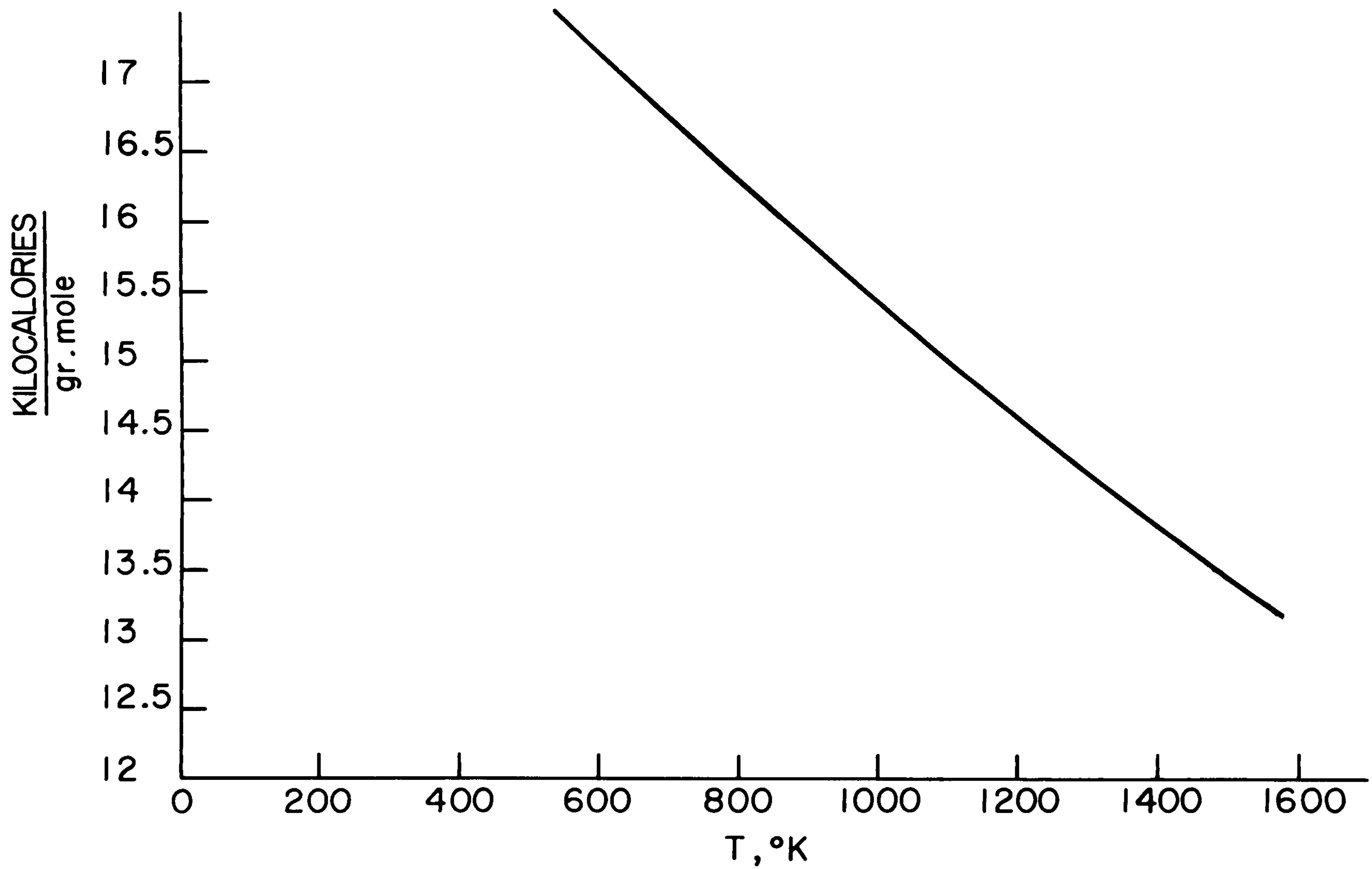


Figure 13. Heat of Vaporization vs Temperature for Cesium (Data From Reference 4)

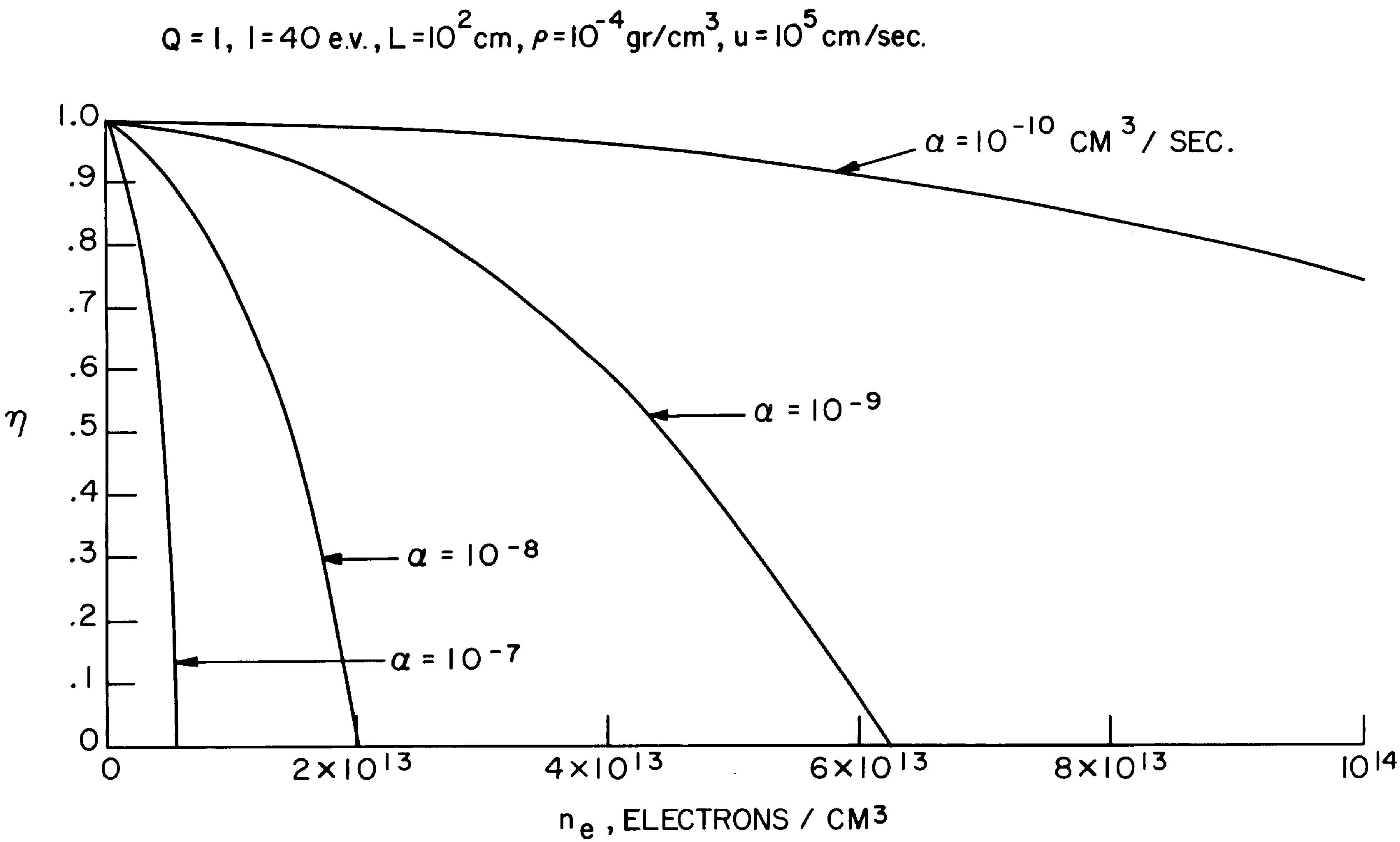


Figure 14. Electron Beam Efficiency vs Plasma Electron Density for Various Radiative Recombination Coefficients